NONSTATIONARY HEATING OF A FIXED GRANULAR MASS

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The reduction of the system of heat-transfer equations in phases of a disperse medium to a single equation is considered. The problem of heating a layer of granular material by a stream of hot fluid is investigated as an illustration.

The regularities of heat transfer between granular material particles and the flux of a continuous medium are of significant applied interest in connection with processes of heat treatment of items in granular heat-carrier layers, of drying and roasting dispersed materials, of chemical reactor operation, and of other apparatus with a fixed or fluidized granular bed, as well as in connection with problems of mastering geothermal resources, producing thermal methods of acting on oil-bearing strata, etc.

Mathematical modeling of heat-transport processes in disperse media and the production of engineering methods for their analysis under different specific conditions are made difficult both by the lack of a sufficiently representative general physical model and uncertainties manifested in applying known partial variants of the model and by the fact that the results obtained using them as a basis are ordinarily quite awkward and do not, by far, always allow simple interpretation. Hence, in addition to the general problem of developing and perfecting the physical model itself, the problem also occurs of investigating the applicability conditions of the partial variants and of their further simplification. One of the widespread methods of describing the heat transfer in a disperse medium is considered below in such a context.

For simplicity we limit ourselves to an investigation of the heat transfer in a fixed granular mass in which a fluid is filtered with a filtration velocity $\varepsilon\mathbf{u}$ and the contact heat conduction over the body formed by the particles is neglected. We write the heat-transfer equation in phases in a continual approximation in "quasistationary" form:

$$\varepsilon d_0 c_0 \left(\frac{\partial}{\partial t} + \mathbf{u}\nabla\right)\tau_0 = \lambda\Delta\tau_0 - \beta(\tau_0 - \tau_1),\quad (1-\varepsilon)\, d_1 c_1 \frac{\partial \tau_1}{\partial t} = \beta(\tau_0 - \tau_1). \tag{1}$$

The coefficients λ and β are here considered independent of the coordinates and time. Equations of type (1) were quite frequently used on an empirical basis; they were introduced in the domestic literature, e.g., in [1-3], and they were obtained rigorously on the basis of averaging over the volume and the ensemble in [4, 5]. Neglecting the contact heat conduction under conditions ordinarily encountered is fully justified as can be seen from numerous experiments as well as from an analysis of such conduction and thermal resistance of a single contact in [6, 7]. Radiation heat transfer is not taken into account in (1), which limits the analysis to sufficiently low temperatures.

The independence of λ and β from the coordinates assumes, firstly, the structural homogeneity of the granular material. Secondly, when the Reynolds and Péclet numbers for a single particle are large compared to one so that convective heat dispersion in the intersected pore space of the material and the convective heat flux on the particle play a part, homogeneity of the filtration flux is also required for this. The time independence of λ and β assumes that the characteristic time of a substantial change in the mean phase temperatures τ_0, τ_1 is considerably greater than the inner and outer local relaxation times of the temperature fields outside and inside a single particle. These times agree with $a^2 d_0 c_0/\lambda_0$, and $a^2 d_1 c_1/\lambda_1$, respectively, in order of magnitude for low Reynolds and Péclet numbers. As these numbers increase, the relaxation times for a layer of particles of this size diminish monotonically (see [8], for example). If the last condition is not satisfied, then both the frequency dispersion of the effective heat conduction λ and the time dependence of the Nusselt number for a particle, i.e., the dispersion of the effective coefficient of interphasal heat transfer β investigated in [9], are essential so that the quasistationary formulation considered here ceases to be true.

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The solution of different boundary-value problems for (1), if it is generally successfully obtained, will ordinarily yield results difficult to see. Hence, it is desirable to reduce the investigation of system (1), first, to the analysis of a certain equation for a single dependent variable. Such a program is actually formulated by N. V. Antonishin and his co-workers (see [10], for example). Paper [9] is analogous in nature.

To obtain such an equation it is sufficient to express the quantity τ_1 in terms of τ_0 in the general operator form from the second equation in (1) and to use the formal expansion of the operator in a Taylor series. We obtain

$$\tau_1 = \frac{1}{1+\alpha_t\partial/\partial t}\tau_0 = \sum_{n=0}^{\infty}(-1)^n\alpha_t^n\frac{\partial^n\tau_0}{\partial t^n}, \quad \alpha_t = \frac{(1-\varepsilon)\,d_1c_1}{\beta}, \tag{2}$$

where α_t is the "natural" time scale of the heat-conduction process in a two-phase dispersed medium. Substitution of (2) into the first equation in (1) results in obtaining the desired equation which, however, contains an infinite chain of time derivatives of arbitrarily high order.

It is convenient to introduce a "natural" linear scale α_x and dimensionless variables and parameters in conformity with the equalities

$$t = \alpha_t T,\ r = \alpha_x R,\ \alpha_x = \sqrt{\frac{\lambda}{\beta}},\quad U = \frac{d_0c_0}{\sqrt{\lambda\beta}}\varepsilon u,\ \gamma = \frac{\varepsilon}{1-\varepsilon}\frac{d_0c_0}{d_1c_1} \tag{3}$$

in addition to the α_t. System (1) then takes the form

$$\gamma\frac{\partial\tau_0}{\partial T} + U\nabla_R\tau_0 = \Delta_R\tau_0 - (\tau_0-\tau_1),\quad \frac{\partial\tau_1}{\partial T} = \tau_0-\tau_1, \tag{4}$$

and the "equivalent" equation obtained is again written in the form

$$(1+\gamma)\frac{\partial\tau_0}{\partial T} + U\nabla_R\tau_0 = \sum_{n=2}^{\infty}(-1)^n\frac{\partial^n\tau_0}{\partial T^n} + \Delta_R\tau_0. \tag{5}$$

Different approximate models correspond to retaining a different number of time derivatives in (5). Thus, neglecting all such derivatives (the zero approximation) is possible only for the description of the stationary heat-transfer process. The next, first, approximation results in the usual single-phase parabolic equation of convective heat conduction, in which the effective specific heat equals the mean specific heat of the granular material filled with a fluid. This approximation evidently corresponds to an assumption about the instantaneous equalibration of the phase temperatures. Such a single-temperature model was considered in [11, 12], for example. The second approximation results in an elliptic equation for τ_0 (the possibility of the appearance of elliptic equations in heat-transport processes in a dispersed medium was apparently first noted in [9]) etc. In contrast to the zero and first approximations, the second approximation takes account of the difference in the phase temperatures and, therefore, corresponds to a two-temperature model of a dispersed medium.

An operator expansion of type (2) has been used earlier in reports on the construction of simplified rheological models of non-Newtonian media (see [13], for instance) and has meaning only if the time scale ω^{-1} of the quantity τ_0 is very much greater than the natural scale α_t. In fact, upon replacing the operators $\partial^n/\partial t^n$ in (2) by ω^n, it is easy to see that the series obtained has a finite radius of convergence $\omega = \alpha_t^{-1}$. Therefore, the condition of applicability, in principle, of the approximations discussed above reduces to compliance with the inequality $\omega \ll \alpha_t^{-1}$ for the characteristic frequency of the real heat-transfer process.

For a small Péclet number, the steady value of the heat elimination coefficient for a single particle is on the order of λ/a; the number of particles per unit volume is proportional to $(1-\varepsilon)/a^3$. Hence, we have $\beta \sim (1-\varepsilon)\lambda/a^2$. Using this in the expression for α_t from (2), we see that α_t has the same order of magnitude as one or even both the relaxation times mentioned above. (In this case $\alpha_t t^{-1}$ is the inverse to the Fourier number usually introduced. However, this is not so, e.g., in the case when the Péclet number is large, i.e., convective heat transfer to the particle dominates.) Hence, the condition $\omega \ll \alpha_t^{-1}$ is equivalent to the condition for the validity of the quasistationary model itself in (1), and if this model is legitimate, then the analysis of its mentioned approximations is equally legitimate.

The connection between the solutions of the "exact" equations (4) and the different approximations obtained from (5) is investigated below in an example of the one-dimensional problem on the heating of a semi-infinite granular mass, which is of independent interest. A fluid flow with temperature τ_* flows into a mass occupying the half space $x > 0$; the initial temperature of the mass is zero. For simplification, the boundary condition of the first kind is simply taken for $x = 0$, i.e., heat transfer in the domain $x < 0$ is not considered.

For such a problem system (4) has the form

$$\gamma \frac{\partial \tau_0}{\partial T} + U \frac{\partial \tau_0}{\partial X} = \frac{\partial^2 \tau_0}{\partial X^2} - (\tau_0 - \tau_1), \quad \frac{\partial \tau_1}{\partial T} = \tau_0 - \tau_1, \tag{6}$$

and Eq. (5), in which only N principal time derivatives remained, is written in the form

$$(1+\gamma)\frac{\partial \tau_0}{\partial T} + U\frac{\partial \tau_0}{\partial X} = \sum_{n=2}^{N} (-1)^n \frac{\partial^n \tau_0}{\partial T^n} + \frac{\partial^2 \tau_0}{\partial X^2}. \tag{7}$$

The boundary conditions for (6) and (7) are written identically:

$$\tau_0 = \tau_*, \ X = 0, \ T \geqslant 0; \ \tau_0 \to 0, \ X \to \infty, \ T \geqslant 0. \tag{8}$$

The initial conditions for the system (6) are also of standard form:

$$\tau_0 = \tau_1 = 0, \ X > 0, \ T = 0. \tag{9}$$

The initial conditions for (7) follow formally from (2) and (9):

$$\tau_0 = \frac{\partial \tau_0}{\partial T} = \cdots = \frac{\partial^{N-1} \tau_0}{\partial T^{N-1}} = 0, \ X > 0, \ T = 0. \tag{10}$$

Moreover, it is sometimes more convenient to use the physically evident condition

$$\lim_{T \to \infty} \tau_0 = \tau_*, \ X \geqslant 0. \tag{11}$$

For simplification of the calculations, the degree of approximation realized by the solution of the problem for (7) to the solution of (6) will be considered separately for the cases $U \ll 1$ and $U \gg 1$ (conductive or convective heat transfer predominates) for $\gamma \approx 0$ (which corresponds to a gas flow). Let us apply the Laplace transform in variable T. The transform of the solution of problem (5), (8), (9) for $U \ll 1$ has the form

$$\frac{\hat{\tau}_0}{\tau_*} = \frac{1}{p}\exp\left(-X\sqrt{\frac{p}{1+p}}\right) = \frac{1}{p}\sum_{n=0}^{\infty}\frac{(-1)^n}{n!}\left(\frac{p}{1+p}\right)^{n/2} X^n, \tag{12}$$

so that a formal representation follows for the original in the form of a series which can turn out to be convenient for small X:

$$\frac{\tau_0}{\tau_*} = 1 - \exp\left(-\frac{T}{2}\right)\left[XI_0\left(\frac{T}{2}\right) - \frac{1}{\sqrt{T}}\sum_{n=2}^{\infty}\frac{(-1)^n}{n!} X^n M_{\frac{n-1}{2},0}(T)\right], \tag{13}$$

where $I_0(x)$ is the Bessel function of imaginary argument and $M_{\nu,0}(x)$ is the Whittaker degenerate hypergeometric function.

For values of $T \gg 1$, which are only of interest in the context of this work, expansion (12) in a series in p must be used:

$$\frac{\tau_0}{\tau_*} = \frac{\exp(-X\sqrt{p})}{p}\left[1 + X\sqrt{p}\left(\frac{p}{2} - \frac{3p^2}{8} + \cdots\right) + \frac{(X\sqrt{p})^2}{2}\left(\frac{p}{2} - \frac{3p^2}{8} + \cdots\right)^2 + \cdots\right]. \tag{14}$$

The solution of problem (7), (8), (10), for the parabolic equation (the series in (7) is generally discarded) results in (14), in which all the terms in the square brackets are replaced by one. The solution of the same problem for an elliptic equation (only the first term of the series in (7) is retained) also yields an expression of the type (14) in which the coefficient $-3/8$ for p^2 in the parentheses is replaced by $1/8$. Therefore, terms proportional to $(X\sqrt{p})^n p^m$, where $n = 0, \ldots,$ $m \geq n$, are in expansions (14) in the square brackets, for all three problems under consideration. In the first approximation (the parabolic equation) only the term with $n = 0$, $m = 0$ remains; consequently, the condition for applicability of this approximation has the form $Xp^{3/2} \ll 1$, which corresponds to the inequality $T \gg X^{2/3}$. The second approximation (elliptic equation) yields correct values for all terms of the form $(X\sqrt{p})^n p^n$. Hence, as is easy to see, $p \ll 1$ will be the condition for its applicability, which corresponds to $T \gg 1$. Therefore, in contrast to the problem for the heat-conduction parabolic equation, which approximates the solution of the exact problem nonuniformly in X, the problem for the elliptic equation approximates it uniformly, whereby this is the fundamental advantage of the second approximation of the problem over the first. The next approximations [$N > 2$ in (7)] refine the second somewhat for $T \gg 1$ but introduce nothing new in principle. Let us note that $T \gg 1$ is the condition for the adequacy of the formulation of the physical problem in quasistationary form; hence, the solution of the problem for the elliptic equation is not worse than the solution of the exact problem. All the terms in the parentheses, except the first, can be neglected in evaluating the original from (14). The first two terms of the series for the original have the form

$$\frac{\tau_0}{\tau_*} = \operatorname{erfc}\frac{\eta}{2} + \frac{\eta}{4\sqrt{\pi T}}\left(\frac{\eta^2}{2} - 1\right)\exp\left(-\frac{\eta^2}{4}\right) + \cdots, \quad \eta = \frac{X}{\sqrt{T}}. \tag{15}$$

It is easy to write explicit expressions for even the succeeding terms of the series (15) which are of high order in powers of T^{-1} by using formulas for the inversion of the Laplace transform. The exact solution of the problem for the parabolic equation is expressed by the first term of this series. Use of series (15) is evidently convenient only for $T \gg 1$, $\eta \lesssim 1$.

It is easy to arrive at analogous deductions by considering the problem in the case $U \gg 1$ also. The analog of (13) has the form

$$\frac{\tau_0}{\tau_*} = 1 - \exp(-T)\sum_{n=1}^{\infty}\frac{(-1)^n}{n!}\left(\frac{X}{U}\right)^n L_{n-1}(T), \tag{16}$$

where $L_n(x)$ is the Laguerre polynomial. (Let us note that for $\gamma \approx 0$ it is impossible to impose both conditions simultaneously in (10); the second condition in (10) was used in obtaining (16), just as in (13).) Summing the series in (16) by using the definition of one of the generating functions for the Laguerre polynomials, we obtain after evaluation

$$\frac{\tau_0}{\tau_*} = 1 - \exp(-T)\int_0^{X/U}\exp(-z)\, I_0(2\sqrt{Tz})\, dz. \tag{17}$$

This is one of the possible modes of writing the solution of the problem under consideration, which is investigated in [14-18], for example. Let us note that the characteristic linear scale

$$\alpha_x' = U\alpha_x = d_0 c_0 \varepsilon u/\beta \tag{18}$$

is in this case independent certainly of λ and considerably greater, for $U \gg 1$, than α_x.

The first approximation to problem (7), (8), (10) yields the trivial result

$$\frac{\tau_0}{\tau_*} = \begin{cases} 1, & X/U < T, \\ 0, & X/U > T. \end{cases} \tag{19}$$

The second corresponds to the problem for the parabolic equation in which the space and time variables change places (as compared with the usual single-phase heat-conduction equation). Supplementing (7), (8), and (10) by condition (11) for $N = 2$, we obtain by using the Laplace transform in the variable X

$$\frac{\tau_0}{\tau_*} = 1 - \frac{1}{2}\left[\operatorname{erfc}\frac{1}{2}\left(\frac{T}{\sqrt{\xi}} - \sqrt{\xi}\right) + \exp(T)\operatorname{erfc}\frac{1}{2}\left(\frac{T}{\sqrt{\xi}} + \sqrt{\xi}\right)\right], \quad \xi = \frac{X}{U}. \tag{20}$$

As before, in the case under consideration it is easy to show that (20) approximates the solution of the exact problem for $T \gg 1$, while (19) is just for $T \gg \sqrt{X}/U$. The simple solution (20) only turns out to be just as accurate as the solution of the initial problem (6), (8), (9) presented in a great number of papers (see [14-18], for example), within the limits of the accuracy in formulating the physical problem. The equivalent parabolic equation [(7) for $U \gg 1$ and $N = 2$] was obtained earlier by Smirnova [19] by another method. A solution of the type (20) was obtained by it and a comparison with certain exact solutions was performed.

Therefore, the solution of system (6) or (4) can successfully be replaced by the solution of (7) or (5) in the general case, in which only the first term in the series of time derivatives is retained. It is easy to see that this permits a significant simplification of the solution of different boundary-value problems describing heat transfer in a dispersed medium, especially those in which the domain of the solution has a complex shape, with complex boundary conditions given in its boundary, or the problem of the heat-conduction equation must be solved additionally outside the domain mentioned with boundary conditions of the fourth kind given on the boundary.

We now obtain a representation of the solution of the specific problem considered here in the general case when $\gamma \neq 0$, $U \sim 1$. Introducing the function φ,

$$\tau(X, T) = \tau_* \exp\left(\frac{UX}{2} + \frac{(1+\gamma)T}{2}\right)\varphi(X, T), \tag{21}$$

we obtain the following equation for φ from (7) for N = 2:

$$\frac{\partial^2\varphi}{\partial T^2} + \frac{\partial^2\varphi}{\partial X^2} - \frac{(1+\gamma)^2 + U^2}{4}\varphi = 0. \tag{22}$$

The conditions imposed on the solution of this equation have the form

$$\varphi = 0,\ T = 0, \text{ and } X \to \infty,\ \varphi = \exp\left(-\frac{(1+\gamma)T}{2}\right),\ X = 0, \text{ and } T \to \infty. \tag{23}$$

Let us apply the Fourier sine transform

$$\varphi(X, T) = \int_0^\infty \Phi(X, \omega)\sin\omega T d\omega. \tag{24}$$

The solution of the equation obtained from (22) by substituting (24), and which satisfies the condition from (23) for $X \to \infty$, is written in the form

$$\Phi(X, \omega) = C(\omega)\exp\left(-X\sqrt{\omega^2 + \frac{(1+\gamma)^2 + U^2}{4}}\right), \tag{25}$$

where it follows from the condition from (23) for X = 0 that

$$C(\omega) = \frac{2}{\pi}\int_0^\infty \exp\left(-\frac{(1+\gamma)T}{2}\right)\sin\omega T dT = \frac{2}{\pi}\,\frac{\omega}{\omega^2 + (1+\gamma)^2/4}. \tag{26}$$

Therefore, the solution of problem (22), (23) has the form

$$\varphi = \frac{2}{\pi}\int_0^\infty \exp\left(-X\sqrt{\omega^2 + \frac{(1+\gamma)^2 + U^2}{4}}\right)\frac{\omega\sin\omega T d\omega}{\omega^2 + (1+\gamma)^2/4}. \tag{27}$$

Unfortunately, the integral in (27) is not expressed in terms of known functions, but it converges rapidly and can, consequently, be estimated easily numerically. The quantity τ_0 is determined by the relationship (21), while τ_1 can be found from the relationship

$$\tau_1 = \tau_0 - \frac{\partial\tau_0}{\partial T} + \frac{\partial^2\tau_0}{\partial T^2}, \tag{28}$$

which follows from (2) in the approximation under consideration.

Let us note that the solution obtained in the approximation mentioned will satisfy the first but not the second initial condition in (10), whose place is taken by condition (11) in this case. This is related to the specifics of the approximate boundary-value problem under consideration for the elliptic type equation and will result, as is easily shown, in an error on the order of T^{-1}, which is insignificant for $T \gg 1$. In particular, the initial temperature τ_1 evaluated from (28) will differ slightly from zero.

For $U \ll 1$ an explicit expression for the heat flux is easily found from (27). Taking into account that $\tau_0 \approx \tau_*\varphi\exp[(1+\gamma)T/2]$, we obtain

$$-\lambda\frac{\partial\tau_0}{\partial x} = \frac{\lambda\tau_*}{\alpha_x}\exp\left(\frac{(1+\gamma)T}{2}\right)\frac{2}{\pi}\int_0^\infty \exp\left(-X\sqrt{\omega^2 + \frac{(1+\gamma)^2}{4}}\right)\frac{\omega\sin\omega T d\omega}{\sqrt{\omega^2 + (1+\gamma)^2/4}} =$$
$$= \frac{\lambda\tau_*(1+\gamma)}{\pi\alpha_x}\,\frac{T}{\sqrt{T^2 + X^2}}\exp\left(\frac{(1+\gamma)T}{2}\right)K_1\left(\frac{1+\gamma}{2}\sqrt{T^2 + X^2}\right), \tag{29}$$

where $K_1(x)$ is the Macdonald function. Relationship (29) is useful in the respect that it permits rapid estimation of the quantity of heat being absorbed by the granular material in different areas of the mass. An alternate integral representation for φ with $U \ll 1$ also follows from (29) (the formula for $\partial\varphi/\partial X$ is considered as a differential equation for φ with the obvious initial condition for X = 0):

$$\varphi = \exp\left(-\frac{(1+\gamma)T}{2}\right) - \frac{1+\gamma}{\pi} T \int_0^X K_1\left(\frac{1+\gamma}{2}\sqrt{T^2+X^2}\right)\frac{dX}{\sqrt{T^2+X^2}}. \quad (30)$$

For $U \gg 1$ the exponential in (27) can be written approximately in the form $\exp(-XU/2)$, after which result (20) already obtained can easily be reproduced by integration.

In conclusion, let us make two remarks about the prospects for further research in the direction considered. Firstly, the assumption usually used in modeling that convective heat transfer predominates in the presence of filtration turns out to be not so obvious by far for real filtration rates as it is ordinarily assumed to be. Meanwhile, even the type of solution changes in going from $U \gg 1$ to $U \ll 1$. Hence, the heat-transport problem in infiltrable (and also porous generally) masses must be investigated in greater detail for intermediate values of U. This remark is especially important for heat transfer in slightly permeable strata customary for the use of geothermal resources, the heating of oil-bearing strata, etc.

Secondly, the initial nonstationary stage of a process corresponding to the inequality $T \leq 1$ is essential for a number of problems on the heating of a granular layer or items submerged therein (e.g., on the heating of a "packet" making contact with the surface in a fluidized bed). Hence, it is quite important to investigate the influence of a local nonstationarity, i.e., the time dependence of the quantities λ and β for medium heat transfer, whose effective solution was acknowledged necessary in [16, 18], e.g., as well as [9]. This remark refers to an equal degree to the need to take account of the presence of a layer of elevated porosity adjoining the solid surface to be placed in the granular material. The latter is usually done by using the semiempirical insertion of the coefficient of "contact" resistance to heat flux [20]. However, it is easy to see that it is impossible to consider this coefficient a quantity independent of time in a substantially nonstationary heat-transport process.

NOTATION

a	is the particle radius;
C	is the quantity defined in (26);
c	is the specific heat;
d	is the density;
p	is the Laplace transform parameter;
r, R	are the dimensional and dimensionless radius-vectors;
t, T	are the dimensional and dimensionless times;
U	is the parameter introduced in (3);
u	is the mean velocity in the gaps between particles;
x, X	are the dimensional and dimensionless coordinates;
α_t, α_x, α_x'	are the time and linear scales;
β	is the coefficient of interphasal heat transfer;
γ	is the parameter introduced in (3);
ε	is the porosity;
$\eta = X/\sqrt{T}$; λ	is the heat conduction;
$\xi = X/U$; τ	is the temperature;
ω	is the characteristic frequency of the heat-transport process and the Fourier transform parameter;
φ, Φ	is the function defined in (21) and its Fourier sine transform; the subscripts zero and one refer to the continuous and dispersed phases, respectively, and the upper caret denotes the Laplace function transform.

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FLOW AND HEAT TRANSFER IN COAXIAL JET FLOW AROUND AN OBSTACLE

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Results are presented of an experimental investigation of the gasdynamics and heat transfer in the reverse flow zone near an obstacle during coaxial jet flow on it along the normal.

In connection with the possibility of a directional influence on the nature of the flow and heat transfer at the surface of streamlined bodies, a considerable interest has recently been manifested in the problem of interaction between nonuniform flows of the "wake" type and blunt bodies placed across the stream [1-5]. It is experimentally shown in [2, 3] that a stable circulation flow with reverse currents to the central point of the body can be realized near the body for definite values of the ratio between the stream velocity at the circumference and the velocity in the central part (the coflow parameter is $m = U_2/U_1 > 1$).

The authors posed the problem of studying the effect of the origin of the return currents zone for jet flow around the obstacle and the possibility of its practical application for the intensification of heat transfer in the area of jet interaction with obstacles. The investigation, on the whole, is experimental in nature and is a continuation of [2], in which are presented preliminary results on the interaction between a subsonic axisymmetric jet with circumferential maximum velocity at the nozzle exit and a plane obstacle.

The experimental investigations were performed on an apparatus [6] consisting of a wind tunnel to whose stilling chamber an axisymmetric contractor representing a Vitoshinskii nozzle with waisting 9 and exit diameter $d_2 = 100$ mm is fastened, and a two-stage coordinating unit with measuring obstacles thereon. To obtain coaxial jets in the nozzle, an additional central contractor with exit diameter $d_1 = 25, 50, 75$ mm is inserted along its axis. Variation of the cojet parameter m assured the mounting of interchangeable grids with different clogging coefficients in the central contractor.

The flow and heat transfer were studied in the interaction domain by using "dynamic" and "thermal" plane obstacles permitting the measurement of the static pressure and the heat-transfer coefficient α on the obstacle surface, as well as the longitudinal component of the average velocity in the interaction domain near the obstacle. The static pressure on the obstacle was measured by drainage of the "dynamic" obstacle with a 1-mm-diameter collector hole. The transducer DD-6, operating in the range to 0.4 bar, was used as pressure sensor in conjunction with the measuring apparatus VI6-5MA and recording on an N-117 loop oscillograph during continuous pulling of the obstacle across the jet with the distance tied to markers in the path. The error in determining the pressure did not exceed 2%.

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